Adaptive estimation of heteroskedastic error component models

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Abstract

This paper checks the sensitivity of two adaptive heteroskedastic estimators suggested by Li and Stengos (1994) and Roy (2002) for an error component regression model to misspecification of the form of heteroskedasticity. In particular, we run Monte Carlo experiments using the heteroskedasticity set up of Li and Stengos (1994) to see how the misspecified Roy (2002) estimator performs. Next, we use the heteroskedasticity set up of Roy (2002) to see how the misspecified Li and Stengos (1994) estimator performs. We also check the sensitivity of these results to the choice of the smoothing parameters, the sample size and the degree of heteroskedasticity.

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1 Introduction

Li and Stengos (1994) derived an adaptive estimator for the heteroskedastic error component model using panel data. The heteroskedasticity was placed on the remainder error term, see also Baltagi (1988) and Wansbeek (1989). More recently, Roy (2002) derived a similar adaptive estimator for the heteroskedastic error component model where the heteroskedasticity was placed on the individual specific error rather than the remainder disturbance, see also Mazodier and Trognon (1978) and Baltagi and Griffin (1988). These adaptive methods are promising since they rely on nonparametric methods which are robust to functional misspecification (see Carroll (1982), Robinson (1987), Rilstone (1991), Delgado (1992), Hidalgo (1992) to mention a few papers in the non panel literature on adaptive estimation in the presence of heteroskedasticity). These adaptive estimation methods are also well suited for micro panels since they only require large N asymptotics and can handle T small or fixed. This is important because estimation of general heteroskedastic error component models (without any structure on the variances) with small T is not advisable, see Baltagi (2001). Essentially because one is estimating N variances (N large) with NT observations (T small).

Both Li and Stengos (1994) and Roy (2002) run Monte Carlo simulations for their respective models comparing their adaptive estimators, which account for heteroskedasticity, with OLS, fixed effects and random effects estimators all of which ignore heteroskedasticity. Li and Stengos (1994) find that their adaptive estimator outperforms all the other estimators for $N = 50$ and $T = 3$ and for moderate to severe degrees of heteroskedasticity. Roy (2002) also finds that her adaptive estimator performs well, although it was outperformed by fixed effects in some cases where there were moderate and severe degrees of heteroskedasticity. Roy also reports that her estimator was sensitive to the choice of the smoothing parameter.

This paper checks the sensitivity of the two proposed adaptive heteroskedastic estimators under misspecification of the form of heteroskedasticity. In particular, we run Monte Carlo experiments using the heteroskedasticity set up of Li and Stengos (1994) to see how the misspecified Roy (2002) estimator performs. Next, we use the heteroskedasticity set up of Roy (2002) to see how the misspecified Li and Stengos (1994) estimator performs. We also check the sensitivity of these results to the choice of the smoothing parameters, the sample size and the degree of heteroskedasticity.

2 The model

Consider the standard regression model with one way error component disturbances:

$$y_{it} = x_{it}\beta + u_{it} \ , \ u_{it} = \mu_i + v_{it} \ , \ i = 1, ..., N \ , \ t = 1, ..., T \tag{1}$$

where x_{it} is $1 \times k$. For the Li and Stengos (1994) model $\mu_i \sim IID\left(0, \sigma_\mu^2\right)$ and $E\left[v_{it}|x_{it}\right] = 0$ with $Var\left[v_{it}|x_{it}\right] = \gamma\left(x_{it}\right) \equiv \gamma_{it}$. So that the heteroskedasticity is on the remainder error term (see also Baltagi (1988) and Wansbeek (1989)).

Therefore

$$\sigma_{it}^2 = E\left[u_{it}^2|x_{it}\right] = \sigma_\mu^2 + \gamma_{it} \tag{2}$$

the proposed estimator of σ_μ^2 is given by:

$$\widehat{\sigma}_\mu^2 = \frac{\sum\limits_{i=1}^{N}\sum\limits_{t \neq s}^{T} \widehat{u}_{it}\widehat{u}_{is}}{NT\left(T-1\right)} \tag{3}$$

where $\widehat{u}_{it}$ denotes the OLS residual. Also

$$\widehat{\gamma}_{it} = \frac{\sum\limits_{j=1}^{N}\sum\limits_{s=1}^{T} \widehat{u}_{js}^2 K_{it,js}}{\sum\limits_{j=1}^{N}\sum\limits_{s=1}^{T} K_{it,js}} - \widehat{\sigma}_\mu^2 \tag{4}$$

where the kernel function is given by:

$$K_{it,js} = K\left(\frac{x_{it} - x_{js}}{h}\right) \tag{5}$$

and h is the smoothing parameter. These estimators of the variance components are used to construct a feasible adaptive GLS estimator of β which they denote by GLSAD. The computation of their feasible GLS estimator is simplified into an OLS regression using a recursive transformation that reduces the general heteroskedastic error components structure into classical errors, see Li and Stengos (1994) for details. Li and Stengos (1994) choose the normal kernel and the smoothing parameter is set as:

$$h = c.s_x.\left(NT\right)^{-1/5} \tag{6}$$

where c is a constant, s_x is the sample standard deviation of x_{it} and NT is the sample size. Li and Stengos (1994) computed h using $c = 0.8, 1$ and 1.2. Since the results were similar, they only reported the results for $c = 1$.

For the Roy (2002) model, $E\left[\mu_i|\overline{x}_{i.}\right] = 0$ with $Var\left[\mu_i|\overline{x}_{i.}\right] = \omega\left(\overline{x}_{i.}\right) \equiv \omega_i$ with $\overline{x}_{i.} = \sum\limits_{t=1}^{T} x_{it}/T$ and $v_{it} \sim IID\left(0, \sigma_v^2\right)$. So that the heteroskedasticity is on the individual specific error component (see also Mazodier and Trognon (1978) and Baltagi and Griffin (1988)). Roy (2002) used the usual estimator of σ_v^2:

$$\widehat{\sigma}_v^2 = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left[(y_{it} - \overline{y}_{i.}) - \widehat{\beta}_w (x_{it} - \overline{x}_{i.})\right]^2}{N(T-1) - k} \tag{7}$$

where $\widehat{\beta}_w$ is the fixed effects or within estimator of β. Also

$$\widehat{\omega}_i = \frac{\sum_{j=1}^{N}\sum_{t=1}^{T}\widehat{u}_{jt}^2 K_{i.,j.}}{\sum_{j=1}^{N}\sum_{t=1}^{T}K_{i.,j.}} - \widehat{\sigma}_v^2 \tag{8}$$

where the kernel function is given by:

$$K_{i.,j.} = K\left(\frac{\overline{x}_{i.} - \overline{x}_{j.}}{h}\right) \tag{9}$$

Using these estimators of the variance components, Roy (2002) computed a feasible GLS estimator using the transformation derived by Baltagi and Griffin (1988). This was done for various values of $h = 0.5$, 1.0 and 1.5 and was denoted EGLS($h = 0.5$), etc.

3 Monte Carlo results

The design of our Monte Carlo experiment follows closely that of Li and Stengos (1994) and Roy (2002) which in turn adapted it from Rilstone (1991) and Delgado (1992). The following simple regression model is considered:

$$y_{it} = \beta_0 + \beta_1 x_{it} + \mu_i + v_{it}\ ,\ i = 1, ..., N\ ,\ t = 1, ..., T \tag{10}$$

where

$$x_{it} = 0.5 w_{i,t-1} + w_{i,t} \tag{11}$$

We generate $w_{i,t}$ as $i.i.d\ U(0,2)$. The parameters β_0 and β_1 are assigned values 5 and 0.5 respectively. We choose $N = 50, 100$ and $T = 3, 5$ and 10. For each x_i, we generate $T + 10$ observations and drop the first ten observations in order to reduce the dependency on initial values.

Case 1: For the Roy (2002) set up, we generate v_{it} as $i.i.d\ N(0, \sigma_v^2)$ and $\mu_i \sim N(0, \omega_i)$ where

$$\omega_i = \omega(\overline{x}_{i.}) = \alpha^2 (1 + \lambda \overline{x}_{i.})^2 \tag{12}$$

Denoting the expected variance of μ_i by $\overline{\omega}$, Roy (2002) fixed the expected total variance $\overline{\sigma}^2 = \overline{\omega} + \sigma_v^2 = 8$ to make it comparable across the different data generating processes. Following Roy (2002), we let σ_v^2 take the values 2, 4 and 6. For each fixed value of σ_v^2, λ is assigned values 0, 1, 2 and 3 with $\lambda = 0$ denoting the homoskedastic individual specific error. For a fixed value of σ_v^2, we obtain a value of $\overline{\omega} = (8- \sigma_v^2)$ and using a specific value of λ, we get the corresponding value for α from (12).

Case 2: For the Li and Stengos (1994) set up, we generate μ_i as *i.i.d* $N\left(0, \sigma_\mu^2\right)$ and $v_{it} \sim N\left(0, \gamma_{it}\right)$ where

$$\gamma_{it} = \gamma\left(x_{it}\right) = \alpha^2\left(1 + \lambda x_{it}\right)^2 \tag{13}$$

Denoting the expected variance of v_{it} by $\overline{\gamma} = E\left[\gamma\left(x_{it}\right)\right]$, Li and Stengos (1994) set the expected total variance $\overline{\sigma}^2 = \sigma_\mu^2 + \overline{\gamma} = 8$ to make it comparable across the different data generating processes. We let σ_μ^2 take the values 2, 4 and 6, so that $\rho = \sigma_\mu^2/\overline{\sigma}^2 = 0.25, 0.5$ and 0.75. For each fixed value of σ_μ^2, λ is assigned values 0, 1, 2 and 3 with $\lambda = 0$ denoting the homoskedastic remainder error term. For a fixed value of σ_μ^2, we obtain a value of $\overline{\gamma} = (8- \sigma_\mu^2)$ and using a specific value of λ, we get the corresponding value for α from (13).

For each replication, we compute the following estimators: (1) the OLS estimator; (2) the fixed effects or within estimator (Within); (3) the conventional GLS estimator for the one way error component model that assumes the error term u_{it} is homoskedastic (GLSH). We use two feasible GLS estimators of the variance components. One suggested by Wallace and Hussain (1969) which we denote by (GLSH_WH) and the other by Amemiya (1971) which we denote by (GLSH_A); (4) the adaptive heteroskedastic estimator (EGLS) proposed by Roy (2002); and (5) the adaptive heteroskedastic estimator (GLSAD) proposed by Li and Stengos (1994).

For all estimators, 1000 replications are performed and the relative MSE for β_1 is obtained by dividing its estimator's MSE by that of the true GLS. In addition, we obtain the empirical size for each estimator when testing $H_0 : \beta_1 = 0.5$ at the 5% level.

Table 1 reports the results of case 1 (*i.e.*, the Roy (2002) set up). We first look at $N = 50$, $T = 3$ and the homoskedastic error case ($\lambda = 0$). The GLSH estimators perform quite well since they are feasible versions of the best linear unbiased estimator of β_1. In general, the Within estimator performs badly with respect to true GLS especially when σ_v^2 is large. This performance improves as T increases. The OLS estimator performs the worst since it ignores the individual effects and the heteroskedasticity when present. The EGLS performs quite well when $\lambda = 0$, being a close second to GLSH_A for all values of σ_v^2 considered. EGLS (normal h) is computed with the suggested bandwith of Li and Stengos (1994) for a normal kernel (see eq. (6)) where the constant $c = 1$. For $N = 50$ and $T = 3$, this $h = 0.227$. We have also computed EGLS for several h values ($h = 0.5$, $h = 1.0$ and $h = 1.5$). Unlike

Roy (1994), we do not find the performance of EGLS to be very sensitive to h. However, we do find that the misspecified GLSAD of Li and Stengos (1994) to be very sensitive to the choice of h. In fact, GLSAD performs poorly for small values of σ_v^2, small values of h, and large values of λ. However, its performance improves as σ_v^2 or h or T increases. From Table 1, it is clear that EGLS and GLSH are the recommended estimators for various degrees of heteroskedasticity λ and various proportions of the variance due to individual effects (equivalently, various values of σ_v^2). For case 1, misspecifying the form of adaptive heteroskedasticity is costly for GLSAD especially for a small h, a short T or a small σ_v^2.

When T increases, the rankings of the estimators in terms of relative MSE remain the same. GLSH and EGLS estimators outperform the other estimators in Table 1. However, the performance of the Within and GLSAD estimators improve as T increases for all values of σ_v^2 and λ. As the bandwidth of the kernel increases from $h = 0.5$ to 1.5, the GLSAD estimator becomes a viable competitor even though the adaptive form of heteroskedasticity it assumes is misspecified. What is striking in this table is that GLSH, the error component GLS estimator ignoring heteroskedasticity performs just as well as Roy's (2002) EGLS even when λ, the degree of heteroskedasticity, is large. This is true for all values of T and σ_v^2. This confirms similar results obtained by Roy (2002) for GLSH and EGLS in her experiments.

Table 2 replicates the results of Table 1 for $N = 100$ rather than $N = 50$. We see that there are improvements in relative MSE numbers as we go from Table 1 to Table 2 due to doubling the sample size. However, the main conclusions obtained from Table 1 remain intact. EGLS performs well even when $\lambda = 0$ (no heteroskedasticity), but GLSH also performs well even when the degree of heteroskedasticity (λ) is large. The misspecified adaptive heteroskedasticity estimator GLSAD performs badly for small h, small σ_v^2 and large λ. However, its performance improves as h, T or σ_v^2 increases.

Table 3 reports the percentage of cases with negative estimated variances for the adaptive estimators in case 1. The proposed estimators of the variances (eq. (4) and eq. (8)) can lead to negative values depending on the size of the bandwidth h, the degree of heteroskedasticity (λ) and σ_v^2. For example, when $\sigma_v^2 = 2$ (*i.e.*, the remainder variance is 25% of the total variance), the Roy (2002) estimator of the variance (eq. (8)) does not yield any negative variances for all values of h, T and λ. On the other hand, the misspecified GLSAD gives negative variances for small values of h. The percentage of negative variances increases with the degree of heteroskedasticity λ and decreases with T. In fact, GLSAD has the highest percentage of negative variances (15.9%) for $\sigma_v^2 = 2$, small h, $T = 3$ and $\lambda = 3$. When $\sigma_v^2 = 4$ (*i.e.*, the remainder variance is 50% of the total variance), both the Roy (2002) and the Li and Stengos (1994) estimators of the variances (eq. (8) and eq. (4))

yield negative variances for the normal h. However, this occurs in less than 1% of the cases. When $\sigma_v^2 = 6$ (*i.e.*, the remainder variance is 75% of the total variance), the Li and Stengos (1994) estimator of the variance (eq. (4)) is almost free of negative variance estimates. On the other hand, the EGLS estimator yields negative variance estimates in at most 9.6% of the cases for $\sigma_v^2 = 6$, $T = 3$, $\lambda = 3$ and normal h. The percentage of negative variances for EGLS increases with the degree of heteroskedasticity λ and decreases with T and h. A similar table for the percentage of negative variance estimates for case 1 with $N = 100$ is available upon request from the authors.

Table 4 reports the results of case 2 (*i.e.*, the Li and Stengos (1994) set up). We first look at $N = 50$, $T = 3$ and the homoskedastic error case ($\lambda = 0$). As expected the GLSH estimators performs quite well since they are the feasible versions of the best linear unbiased estimator. However, GLSH performs badly as the degree of heteroskedasticity λ increases. The loss in efficiency with respect to true GLS is of the order of 34 to 80% for $\lambda = 3$ for various values of σ_μ^2 and T. The Within estimator performs badly with respect to true GLS especially when σ_μ^2 is small. This performance improves as T increases. The OLS estimator also performs badly since it ignores the individual effects and the heteroskedasticity when present. As the degree of heteroskedasticity increases (from $\lambda = 1$ to $\lambda = 3$), GLSAD performs the best in all cases except for $\sigma_\mu^2 = 6$ and small h where negative variance estimates occur. EGLS performs badly under misspecification of the form of heteroskedasticity and its performance is similar to that of GLSH. As in case 1, EGLS does not seem to be as sensitive to the choice of h as GLSAD. This is true despite the fact that EGLS is dealing with a misspecified form of adaptive heteroskedasticity.

Table 5 replicates the results of Table 4 for $N = 100$ rather than $N = 50$. We see that there are improvements in relative MSE numbers as we go from Table 4 to Table 5 due to doubling the sample size. However, the main conclusions obtained from Table 4 remain intact. GLSAD performs the best when $\lambda > 0$. One exception is when $\sigma_\mu^2 = 6$ and h is small. EGLS and GLSH have a similar performance, but the loss in efficiency can be big for both estimators especially for large λ.

Table 6 reports the percentage of cases with negative estimated variances for the adaptive estimators in case 2. EGLS has the highest percentage of negative variances (24%) when $\sigma_\mu^2 = 2$, h is small, $T = 3$ and $\lambda = 3$. GLSAD has the highest percentage of negative variances (10%) when $\sigma_\mu^2 = 6$, h is small, $T = 3$ and $\lambda = 3$. A similar table for the percentage of negative variance estimates for case 2 with $N = 100$ is available upon request from the authors.

Comparing Tables 1 and 2 with Tables 4 and 5, it is clear that the magnitude of the relative MSE is bigger in the latter Tables. The only exception is

GLSAD (normal h) for $\sigma_v^2 = 2$. This means that the adaptive heteroskedasticity of Li and Stengos (1994) causes more efficiency loss in the estimators considered than that implied by the Roy (2002) form of the adaptive heteroskedasticity. In the Roy set up, EGLS is the best in terms of relative MSE but GLSH is not far behind. The latter estimator ignores the heteroskedasticity completely. GLSAD is also a viable estimator (as long as the choice of h is not too small) even though it is dealing with a misspecified form of heteroskedasticity . For the Li and Stengos set up, GLSAD ranks the best in terms of relative MSE (as long as the choice of h is not too small) followed by GLSH and EGLS . However, the loss in efficiency (relative MSE with respect to true GLS) in case 2 is much larger than that in case 1.

Table 7 reports the results of the 5% size performance of the estimated t-ratios of the slope coefficient for case 1 with $N = 50$. For 1000 replications, counts between 37 and 63 are not significantly different from 50 at the 5% level. Except for OLS and the misspecified GLSAD estimator, the empirical size for all remaining estimators is not significantly different from 5%. OLS and GLSAD (for small h) tend to over-reject the null when true for all values of σ_v^2. This test of hypothesis using EGLS does not seem to be sensitive to the value of the bandwidth. However, this is not the case for the Li and Stengos (1994) GLSAD which is sensitive to the choice of h and leads to over-rejection of the null whenever h is small. A similar table for the empirical size for case 1 when $N = 100$ is available upon request from the authors. Results in Table 7 confirm similar results in Roy (2002) who found that GLSH and EGLS had the correct size no matter what choice of h was used.

Table 8 reports the results of the 5% size performance of the estimated t-ratios of the slope coefficient for case 2 with $N = 50$. Once again, OLS and GLSAD (for small h) tend to over-reject the null when true for all values of σ_μ^2. However, in this case, the other estimators GLSH, EGLS and Within tend to over-reject when the degree of heteroskedasticity λ is large. These rejection rates are in the 7 to 8% range rather than 5%. A similar table for the empirical size for case 2 when $N = 100$ is available upon request from the authors. Li and Stengos (1994) found that only GLSAD had the correct size while OLS, GLSH and Within over-rejected the null hypothesis. Table 8 confirms this result for GLSAD as long as h is not too small.

4 Conclusion

This paper considered two alternative estimators of adaptive heteroskedasticity using a panel data regression model. Our Monte Carlo results show that for the Roy (2002) model, her EGLS estimator is not sensitive to the choice of the bandwidth and performs well in terms of relative MSE. However, this performance is closely matched by GLSH which is the error component GLS ignoring heteroskedasticity. For this model, the misspecified GLSAD estimator of Li and Stengos (1994) shows loss in efficiency especially when a small bandwidth is used. However, this performance improves as the bandwidth increases. For the Li and Stengos (1994) model, their GLSAD ranks the best in relative MSE but is sensitive to the choice of the bandwidth. The misspecified EGLS leads to a big loss in efficiency depending on the degree of heteroskedasticity, the length of the time series and the proportion of the variance due to the individual effects. Even under misspecification, EGLS is not sensitive to the choice of the bandwidth. Its relative MSE performance is again close to that of GLSH. For the test of hypothesis, OLS and GLSAD (small h) tend to over-reject the null when true no matter what form of adaptive heteroskedasticity. In contrast, GLSH, EGLS and Within have size not significantly different from 5% when the true model is that of Roy (2002) and slightly over-reject (7 to 8%) when the true model is that of Li and Stengos (1994).

We conclude that in terms of loss in efficiency, misspecifying the adaptive form of heteroskedasticity can be costly when the Li and Stengos (1994) model is correct and the researcher performs the Roy (2002) estimator. This loss in efficiency is smaller when the true model is that of Roy (2002) and one performs the Li and Stengos (1994) estimator. The latter statement is true as long as the choice of bandwidth is not too small.

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Table 1 - Relative MSE for β_1 in case 1, N=50, 1000 replications

	λ											
	0			1			2			3		
	T=3	T=5	T=10	T=3	T=5	T=10	T=3	T=5	T=10	T=3	T=5	T=10
$\sigma_v^2 = 2$												
OLS	3.0698	4.1135	4.4714	3.1295	4.2694	4.2855	3.1694	4.3440	4.2548	3.1907	4.3827	4.2429
Within	1.0887	1.0332	0.9984	1.0954	1.0355	0.9986	1.0999	1.0368	0.9987	1.1024	1.0375	0.9987
GLSH_A	1.0035	1.0001	0.9992	1.0045	1.0061	0.9994	1.0077	1.0086	0.9995	1.0096	1.0098	0.9995
GLSH_WH	1.0059	1.0003	0.9993	1.0062	1.0071	0.9996	1.0094	1.0098	0.9997	1.0113	1.0112	0.9997
EGLS (normal *h*)	1.0040	0.9971	0.9984	1.0027	1.0004	0.9986	1.0029	1.0014	0.9987	1.0031	1.0018	0.9987
EGLS (*h*=0.5)	1.0035	0.9992	0.9991	1.0024	1.0035	0.9993	1.0035	1.0051	0.9994	1.0042	1.0059	0.9994
EGLS (*h*=1.0)	1.0039	0.9999	0.9992	1.0037	1.0055	0.9994	1.0060	1.0076	0.9995	1.0074	1.0087	0.9995
EGLS (*h*=1.5)	1.0038	1.0000	0.9992	1.0041	1.0059	0.9994	1.0068	1.0083	0.9995	1.0085	1.0095	0.9996
GLSAD (normal *h*)	1.5876	1.7132	1.6104	2.4426	2.0367	2.0077	2.8222	2.6195	1.9138	3.1033	2.6044	2.1222
GLSAD (*h*=0.5)	1.1918	1.1506	1.0638	1.4891	1.4076	1.1303	1.7556	1.5210	1.1523	1.9005	1.5398	1.2124
GLSAD (*h*=1.0)	1.0342	1.0088	1.0030	1.1069	1.0624	1.0033	1.1360	1.1240	1.0052	1.1599	1.1226	1.0064
GLSAD (*h*=1.5)	1.0142	1.0009	0.9984	1.0235	1.0168	0.9976	1.0324	1.0240	0.9978	1.0375	1.0278	0.9980
$\sigma_v^2 = 4$												
OLS	1.5866	1.9469	2.1088	1.6077	2.0121	2.0475	1.6245	2.0420	2.0377	1.6334	2.0575	2.0339
Within	1.2221	1.0813	1.0038	1.2360	1.0847	1.0047	1.2431	1.0868	1.0051	1.2468	1.0879	1.0053
GLSH_A	1.0100	1.0023	0.9986	1.0136	1.0117	0.9993	1.0185	1.0158	0.9996	1.0212	1.0178	0.9998
GLSH_WH	1.0143	1.0033	0.9988	1.0179	1.0136	0.9995	1.0229	1.0179	0.9998	1.0256	1.0201	1.0000
EGLS (normal *h*)	1.0165	1.0001	0.9973	1.0137	1.0046	0.9978	1.0131	1.0060	0.9979	1.0130	1.0068	0.9980
EGLS (*h*=0.5)	1.0124	1.0009	0.9984	1.0122	1.0069	0.9990	1.0138	1.0091	0.9993	1.0147	1.0103	0.9994
EGLS (*h*=1.0)	1.0121	1.0020	0.9987	1.0137	1.0105	0.9993	1.0172	1.0139	0.9996	1.0191	1.0156	0.9997
EGLS (*h*=1.5)	1.0119	1.0024	0.9987	1.0144	1.0115	0.9993	1.0186	1.0153	0.9997	1.0209	1.0173	0.9998
GLSAD (normal *h*)	1.1331	1.1012	1.1043	1.1473	1.1376	1.1061	1.2000	1.1544	1.1002	1.2112	1.1701	1.1072
GLSAD (*h*=0.5)	1.0498	1.0132	1.0064	1.0796	1.0469	1.0073	1.0898	1.0528	1.0084	1.0926	1.0624	1.0090
GLSAD (*h*=1.0)	1.0209	1.0029	0.9988	1.0283	1.0182	0.9989	1.0352	1.0246	0.9993	1.0390	1.0279	0.9995
GLSAD (*h*=1.5)	1.0163	1.0024	0.9983	1.0209	1.0147	0.9985	1.0264	1.0197	0.9988	1.0294	1.0223	0.9990
$\sigma_v^2 = 6$												
OLS	1.1382	1.2545	1.3221	1.1439	1.2832	1.3033	1.1498	1.2955	1.3007	1.1529	1.3017	1.2997
Within	1.4454	1.1712	1.0216	1.4608	1.1732	1.0241	1.4669	1.1748	1.0249	1.4698	1.1756	1.0254
GLSH_A	1.0171	1.0074	0.9981	1.0189	1.0180	0.9994	1.0220	1.0224	1.0001	1.0236	1.0246	1.0004
GLSH_WH	1.0223	1.0096	0.9983	1.0243	1.0211	0.9996	1.0275	1.0258	1.0003	1.0293	1.0281	1.0006
EGLS (normal *h*)	1.0257	1.0140	0.9992	1.0229	1.0171	1.0017	1.0227	1.0173	1.0026	1.0228	1.0176	1.0029
EGLS (*h*=0.5)	1.0215	1.0078	0.9979	1.0205	1.0137	0.9990	1.0211	1.0159	0.9996	1.0214	1.0169	0.9998
EGLS (*h*=1.0)	1.0206	1.0075	0.9981	1.0208	1.0169	0.9994	1.0228	1.0206	1.0000	1.0240	1.0224	1.0003
EGLS (*h*=1.5)	1.0203	1.0081	0.9982	1.0212	1.0184	0.9995	1.0238	1.0226	1.0001	1.0252	1.0246	1.0005
GLSAD (normal *h*)	1.0686	1.0312	1.0246	1.0737	1.0459	1.0269	1.0778	1.0515	1.0280	1.0796	1.0552	1.0286
GLSAD (*h*=0.5)	1.0371	1.0102	1.0015	1.0400	1.0246	1.0027	1.0439	1.0304	1.0034	1.0459	1.0333	1.0038
GLSAD (*h*=1.0)	1.0248	1.0083	0.9983	1.0268	1.0209	0.9995	1.0300	1.0260	1.0001	1.0317	1.0285	1.0005
GLSAD (*h*=1.5)	1.0228	1.0087	0.9982	1.0247	1.0207	0.9994	1.0278	1.0255	1.0000	1.0295	1.0280	1.0004

Table 2 - Relative MSE for β_1 in case 1, N=100, 1000 replications

	λ											
	0			1			2			3		
	T=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10
					$\sigma_v^2 = 2$							
OLS	2.8671	4.2903	4.9044	2.9325	4.2864	4.8601	2.9710	4.3222	4.8706	2.9911	4.3433	4.8781
Within	1.0499	1.0130	1.0115	1.0557	1.0137	1.0133	1.0591	1.0148	1.0139	1.0608	1.0154	1.0142
GLSH_A	1.0043	1.0028	0.9995	1.0118	1.0012	1.0021	1.0157	1.0018	1.0029	1.0177	1.0023	1.0034
GLSH_WH	1.0056	1.0031	0.9994	1.0136	1.0015	1.0019	1.0177	1.0022	1.0028	1.0198	1.0027	1.0032
EGLS (normal *h*)	1.0068	1.0013	0.9998	1.0079	0.9998	1.0005	1.0084	0.9981	1.0008	1.0088	0.9978	1.0009
EGLS (*h*=0.5)	1.0028	1.0015	0.9999	1.0064	0.9990	1.0019	1.0084	0.9989	1.0026	1.0095	0.9990	1.0030
EGLS (*h*=1.0)	1.0038	1.0026	0.9996	1.0099	1.0005	1.0020	1.0131	1.0008	1.0028	1.0148	1.0011	1.0033
EGLS (*h*=1.5)	1.0042	1.0028	0.9995	1.0111	1.0009	1.0020	1.0147	1.0014	1.0029	1.0166	1.0018	1.0033
GLSAD (normal *h*)	1.4187	1.4078	1.5571	1.9216	1.7992	1.5465	2.2468	2.1384	1.5587	2.5245	2.2570	1.6539
GLSAD (*h*=0.5)	1.0739	1.0927	1.0191	1.2169	1.3123	1.0641	1.4030	1.2759	1.0983	1.4867	1.9943	1.1050
GLSAD (*h*=1.0)	1.0132	1.0236	1.0011	1.0446	1.0373	1.0112	1.0515	1.0508	1.0157	1.0567	1.0549	1.0179
GLSAD (*h*=1.5)	1.0079	1.0114	0.9994	1.0192	1.0129	1.0043	1.0250	1.0159	1.0062	1.0280	1.0176	1.0071
					$\sigma_v^2 = 4$							
OLS	1.4975	2.0094	2.2156	1.5279	2.0137	2.2101	1.5454	2.0296	2.2164	1.5544	2.0388	2.2203
Within	1.1486	1.0494	1.0249	1.1592	1.0545	1.0279	1.1646	1.0576	1.0289	1.1674	1.0592	1.0295
GLSH_A	1.0058	1.0044	0.9991	1.0176	1.0057	1.0031	1.0235	1.0082	1.0045	1.0266	1.0097	1.0052
GLSH_WH	1.0062	1.0049	0.9989	1.0184	1.0062	1.0029	1.0245	1.0087	1.0044	1.0277	1.0102	1.0051
EGLS (normal *h*)	1.0114	1.0036	0.9993	1.0132	1.0006	1.0001	1.0141	1.0004	1.0004	1.0146	1.0014	1.0006
EGLS (*h*=0.5)	1.0039	1.0028	0.9996	1.0095	1.0014	1.0028	1.0124	1.0022	1.0039	1.0139	1.0027	1.0045
EGLS (*h*=1.0)	1.0049	1.0042	0.9992	1.0145	1.0043	1.0030	1.0193	1.0062	1.0043	1.0218	1.0073	1.0050
EGLS (*h*=1.5)	1.0055	1.0045	0.9991	1.0163	1.0052	1.0030	1.0218	1.0074	1.0044	1.0246	1.0087	1.0051
GLSAD (normal *h*)	1.0935	1.0821	1.0788	1.1302	1.1112	1.0935	1.1382	1.1242	1.1063	1.1451	1.1373	1.0980
GLSAD (*h*=0.5)	1.0614	1.0267	1.0033	1.0376	1.0331	1.0111	1.0553	1.0397	1.0142	1.0587	1.0434	1.0157
GLSAD (*h*=1.0)	1.0079	1.0125	0.9991	1.0210	1.0148	1.0042	1.0277	1.0182	1.0061	1.0312	1.0201	1.0070
GLSAD (*h*=1.5)	1.0065	1.0083	0.9989	1.0192	1.0098	1.0033	1.0254	1.0125	1.0049	1.0286	1.0142	1.0057
					$\sigma_v^2 = 6$							
OLS	1.0950	1.2755	1.3401	1.1096	1.2828	1.3442	1.1170	1.2905	1.3481	1.1208	1.2948	1.3503
Within	1.3285	1.1288	1.0525	1.3396	1.1387	1.0560	1.3441	1.1428	1.0573	1.3463	1.1449	1.0580
GLSH_A	1.0069	1.0065	0.9983	1.0176	1.0114	1.0032	1.0224	1.0149	1.0050	1.0248	1.0168	1.0058
GLSH_WH	1.0064	1.0074	0.9981	1.0173	1.0122	1.0029	1.0222	1.0158	1.0047	1.0246	1.0177	1.0056
EGLS (normal *h*)	1.0155	1.0119	0.9998	1.0204	1.0146	0.9991	1.0210	1.0143	0.9989	1.0209	1.0139	0.9988
EGLS (*h*=0.5)	1.0059	1.0068	0.9990	1.0116	1.0075	1.0027	1.0140	1.0088	1.0040	1.0152	1.0095	1.0046
EGLS (*h*=1.0)	1.0057	1.0069	0.9985	1.0144	1.0100	1.0030	1.0183	1.0127	1.0047	1.0202	1.0141	1.0055
EGLS (*h*=1.5)	1.0061	1.0071	0.9983	1.0159	1.0111	1.0031	1.0203	1.0142	1.0048	1.0225	1.0158	1.0056
GLSAD (normal *h*)	1.0361	1.0395	1.0278	1.0474	1.0431	1.0341	1.0528	1.0469	1.0365	1.0554	1.0490	1.0376
GLSAD (*h*=0.5)	1.0094	1.0167	1.0010	1.0208	1.0213	1.0066	1.0258	1.0251	1.0086	1.0283	1.0271	1.0095
GLSAD (*h*=1.0)	1.0062	1.0109	0.9983	1.0173	1.0156	1.0035	1.0221	1.0191	1.0053	1.0245	1.0210	1.0062
GLSAD (*h*=1.5)	1.0062	1.0090	0.9981	1.0172	1.0137	1.0031	1.0221	1.0172	1.0050	1.0245	1.0191	1.0058

Table 3 - Percent of negative variances for EGLS and GLSAD estimators in case 1, N=50, 1000 replications

	λ											
	0			1			2			3		
	T=3	T=5	T=10	T=3	T=5	T=10	T=3	T=5	T=10	T=3	T=5	T=10
$\sigma_v^2 = 2$												
EGLS (normal h)	0	0	0	0	0	0	0	0	0	0	0	0
EGLS (h=0.5)	0	0	0	0	0	0	0	0	0	0	0	0
EGLS (h=1.0)	0	0	0	0	0	0	0	0	0	0	0	0
EGLS (h=1.5)	0	0	0	0	0	0	0	0	0	0	0	0
GLSAD (normal h)	6.39	4.02	1.66	12.27	7.88	2.69	14.78	10.13	3.17	15.89	11.15	3.43
GLSAD (h=0.5)	0.93	0.36	0.04	4.53	2.25	0.21	6.37	3.38	0.32	7.26	3.97	0.37
GLSAD (h=1.0)	0	0	0	0.20	0.03	0	0.38	0.08	0	0.49	0.11	0
GLSAD (h=1.5)	0	0	0	0	0	0	0	0	0	0	0	0
$\sigma_v^2 = 4$												
EGLS (normal h)	0.20	0.12	0	0.37	0.14	0.01	0.52	0.19	0.01	0.62	0.23	0.01
EGLS (h=0.5)	0	0	0	0	0	0	0	0	0	0	0	0
EGLS (h=1.0)	0	0	0	0	0	0	0	0	0	0	0	0
EGLS (h=1.5)	0	0	0	0	0	0	0	0	0	0	0	0
GLSAD (normal h)	0.46	0.16	0.04	0.69	0.25	0.06	0.84	0.34	0.06	0.92	0.41	0.07
GLSAD (h=0.5)	0	0	0	0	0	0	0.07	0	0	0.09	0.01	0
GLSAD (h=1.0)	0	0	0	0	0	0	0	0	0	0	0	0
GLSAD (h=1.5)	0	0	0	0	0	0	0	0	0	0	0	0
$\sigma_v^2 = 6$												
EGLS (normal h)	7.36	2.71	0.24	8.6	3.48	0.46	9.24	3.9	0.56	9.55	4.04	0.62
EGLS (h=0.5)	1.83	0.08	0	2.11	0.12	0	2.32	0.15	0	2.4	0.16	0
EGLS (h=1.0)	0.71	0	0	0.69	0	0	0.7	0	0	0.72	0	0
EGLS (h=1.5)	0.56	0	0	0.57	0	0	0.56	0	0	0.56	0	0
GLSAD (normal h)	0.01	0	0	0.01	0	0	0.01	0	0	0.02	0	0
GLSAD (h=0.5)	0	0	0	0	0	0	0	0	0	0	0	0
GLSAD (h=1.0)	0	0	0	0	0	0	0	0	0	0	0	0
GLSAD (h=1.5)	0	0	0	0	0	0	0	0	0	0	0	0

Table 4 - Relative MSE for β_1 in case 2, N=50, 1000 replications

	λ											
	0			1			2			3		
	T=3	*T*=5	*T*=10	*T*=3	*T*=5	T=10	*T*=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10
$\sigma_{\mu}^{2} = 2$												
OLS	1.1382	1.2549	1.3221	1.3789	1.5586	1.7500	1.5206	1.7912	2.0593	1.5975	1.9311	2.2471
Within	1.4454	1.1712	1.0216	1.7290	1.5110	1.4040	1.8977	1.7500	1.6699	1.9893	1.8927	1.8307
GLSH_A	1.0171	1.0074	0.9981	1.2497	1.2941	1.3668	1.3844	1.5014	1.6254	1.4573	1.6255	1.7820
GLSH_WH	1.0223	1.0096	0.9983	1.2547	1.2958	1.3670	1.3894	1.5030	1.6255	1.4624	1.6660	1.7821
EGLS (normal *h*)	1.0257	1.0140	0.9992	1.2737	1.3176	1.3828	1.4115	1.5362	1.6484	1.4861	1.6460	1.8087
EGLS (*h*=0.5)	1.0215	1.0078	0.9979	1.2536	1.3027	1.3673	1.3866	1.5170	1.6260	1.4627	1.6341	1.7826
EGLS (*h*=1.0)	1.0206	1.0075	0.9981	1.2501	1.2953	1.3668	1.3821	1.5036	1.6253	1.4536	1.6282	1.7818
EGLS (*h*=1.5)	1.0203	1.0081	0.9982	1.2510	1.2947	1.3669	1.3844	1.5021	1.6254	1.4566	1.6264	1.7819
GLSAD (normal *h*)	1.0686	1.0312	1.0246	1.1171	1.0780	1.0542	1.1565	1.1088	1.0833	1.1765	1.1491	1.1308
GLSAD (*h*=0.5)	1.0371	1.0102	1.0015	1.0817	1.0811	1.0653	1.1065	1.1372	1.1170	1.1265	1.1740	1.1504
GLSAD (*h*=1.0)	1.0248	1.0083	0.9983	1.1463	1.1664	1.1878	1.2191	1.2876	1.3292	1.1291	1.3616	1.4171
GLSAD (*h*=1.5)	1.0228	1.0087	0.9982	1.1927	1.2230	1.2655	1.2924	1.3820	1.4581	1.3467	1.4780	1.5759
$\sigma_{\mu}^{2} = 4$												
OLS	1.5866	1.9469	2.1088	1.8383	2.2913	2.7314	1.9890	2.5771	3.1757	2.0702	2.7475	3.4427
Within	1.2221	1.0813	1.0038	1.4437	1.3725	1.3720	1.5714	1.5754	1.6265	1.6400	1.6949	1.7793
GLSH_A	1.0100	1.0023	0.9986	1.2095	1.2684	1.3633	1.3228	1.4571	1.6159	1.3836	1.5683	1.7676
GLSH_WH	1.0143	1.0033	0.9988	1.2143	1.2692	1.3633	1.3278	1.4578	1.6159	1.3888	1.5691	1.7676
EGLS (normal *h*)	1.0165	1.0001	0.9973	1.2274	1.2751	1.3613	1.3460	1.4704	1.6137	1.4096	1.5854	1.7653
EGLS (*h*=0.5)	1.0124	1.0009	0.9984	1.2177	1.2711	1.3631	1.3370	1.4625	1.6156	1.3958	1.5755	1.7673
EGLS (*h*=1.0)	1.0121	1.0020	0.9987	1.2132	1.2693	1.3632	1.3270	1.4587	1.6158	1.3881	1.5703	1.7675
EGLS (*h*=1.5)	1.0119	1.0024	0.9987	1.2121	1.2689	1.3633	1.3257	1.4579	1.6159	1.3866	1.5693	1.7676
GLSAD (normal *h*)	1.1331	1.1012	1.1043	1.2330	1.1624	1.1654	1.3342	1.2611	1.3343	1.4313	1.3485	1.4220
GLSAD (*h*=0.5)	1.0498	1.0132	1.0064	1.1059	1.0944	1.0695	1.1430	1.1466	1.1228	1.1515	1.1789	1.1618
GLSAD (*h*=1.0)	1.0209	1.0029	0.9988	1.1249	1.1486	1.1844	1.1861	1.2575	1.3206	1.2194	1.3228	1.4044
GLSAD (*h*=1.5)	1.0163	1.0024	0.9983	1.1625	1.2010	1.2619	1.2466	1.3451	1.4491	1.2921	1.4307	1.5627
$\sigma_{\mu}^{2} = 6$												
OLS	3.0698	4.1135	4.4714	3.4430	4.6556	5.6870	3.6793	5.1610	6.5492	3.8077	5.4631	7.0653
Within	1.0877	1.0332	0.9984	1.2797	1.3009	1.3625	1.3892	1.4876	1.6131	1.4482	1.5971	1.7632
GLSH_A	1.0355	1.0001	0.9992	1.1875	1.2574	1.3631	1.2916	1.4382	1.6137	1.3477	1.5444	1.7638
GLSH_WH	1.0059	1.0003	0.9993	1.1906	1.2573	1.3631	1.2950	1.1438	1.6137	1.3513	1.5443	1.7638
EGLS (normal *h*)	1.0040	0.9971	0.9984	1.1894	1.2553	1.3626	1.2945	1.4366	1.6133	1.3511	1.5431	1.7635
EGLS (*h*=0.5)	1.0035	0.9992	0.9991	1.1883	1.2575	1.3630	1.2928	1.4389	1.6136	1.3491	1.5454	1.7637
EGLS (*h*=1.0)	1.0039	0.9999	0.9992	1.1882	1.2575	1.3631	1.2925	1.4385	1.6137	1.3486	1.5449	1.7638
EGLS (*h*=1.5)	1.0038	1.0000	0.9992	1.1880	1.2574	1.3631	1.2922	1.4383	1.6137	1.3484	1.5446	1.7638
GLSAD (normal *h*)	1.5876	1.7132	1.6104	2.0360	2.0777	1.9566	2.2116	3.1000	2.0659	2.4578	2.6628	2.6016
GLSAD (*h*=0.5)	1.1918	1.1506	1.0638	1.3567	1.2556	1.2456	1.4098	1.4256	1.3613	1.4496	1.4163	1.4505
GLSAD (*h*=1.0)	1.0342	1.0088	1.0030	1.1360	1.1549	1.1852	1.1932	1.2554	1.3200	1.2143	1.3217	1.4033
GLSAD (*h*=1.5)	1.0142	1.0009	0.9984	1.1512	1.1942	1.2611	1.2296	1.3325	1.4463	1.2743	1.4143	1.5583

Table 5 - Relative MSE for β_1 in case 2, N=100, 1000 replications

	λ											
	0			1			2			3		
	T=3	T=5	T=10	T=3	T=5	T=10	T=3	T=5	T=10	T=3	T=5	T=10
$\sigma_\mu^2 = 2$												
OLS	1.0950	1.2755	1.3401	1.3404	1.6097	1.7722	1.4924	1.8298	2.0961	1.5758	1.9562	2.2974
Within	1.3285	1.1288	1.0525	1.6108	1.3926	1.4185	1.7861	1.5792	1.6900	1.8622	1.6872	1.8587
GLSH_A	1.0069	1.0065	0.9983	1.2273	1.2663	1.3474	1.3648	1.4437	1.6077	1.4435	1.5461	1.7696
GLSH_WH	1.0064	1.0074	0.9981	1.2269	1.2679	1.3471	1.3664	1.4456	1.6074	1.4431	1.5481	1.7692
EGLS (normal *h*)	1.0155	1.0119	0.9998	1.2507	1.3232	1.3561	1.3935	1.5147	1.6276	1.4721	1.6237	1.7942
EGLS (*h*=0.5)	1.0059	1.0068	0.9990	1.2281	1.2836	1.3450	1.3690	1.4708	1.6040	1.4469	1.5784	1.7651
EGLS (*h*=1.0)	1.0057	1.0069	0.9985	1.2242	1.2682	1.3466	1.3631	1.4465	1.6065	1.4396	1.5493	1.7681
EGLS (*h*=1.5)	1.0061	1.0071	0.9983	1.2254	1.2673	1.3469	1.3644	1.4450	1.6070	1.4409	1.5475	1.7687
GLSAD (normal *h*)	1.0361	1.0395	1.0278	1.0814	1.0553	1.0387	1.0974	1.0833	1.0607	1.1218	1.1013	1.1027
GLSAD (*h*=0.5)	1.0094	1.0167	1.0010	1.0655	1.0648	1.0714	1.1074	1.1036	1.1323	1.1304	1.1272	1.1734
GLSAD (*h*=1.0)	1.0062	1.0109	0.9983	1.1375	1.1457	1.1844	1.2273	1.2447	1.3332	1.2726	1.3033	1.4285
GLSAD (*h*=1.5)	1.0062	1.0090	0.9981	1.1781	1.1988	1.2552	1.2888	1.3325	1.4529	1.3500	1.4103	1.5774
$\sigma_\mu^2 = 4$												
OLS	1.4975	2.0094	2.2156	1.7748	2.4910	2.8616	1.9366	2.7968	3.3384	2.0240	1.2712	3.6328
Within	1.1486	1.0494	1.0249	1.3563	1.2916	1.3781	1.4820	1.4591	1.6371	1.5502	1.5556	1.7970
GLSH_A	1.0058	1.0044	0.9991	1.1888	1.2486	1.3430	1.3009	1.4139	1.9964	1.3618	1.5089	1.7529
GLSH_WH	1.0062	1.0049	0.9989	1.1896	1.2497	1.3428	1.3019	1.4152	1.5961	1.3628	1.5103	1.7526
EGLS (normal *h*)	1.0114	1.0036	0.9993	1.2054	1.2719	1.3441	1.3242	1.4501	1.5984	1.3886	1.5520	1.7555
EGLS (*h*=0.5)	1.0039	1.0028	0.9996	1.1876	1.2511	1.3428	1.3007	1.4185	1.5959	1.3621	1.5147	1.7522
EGLS (*h*=1.0)	1.0049	1.0042	0.9992	1.1878	1.2494	1.3429	1.3000	1.4151	1.5962	1.3610	1.5104	1.7526
EGLS (*h*=1.5)	1.0055	1.0045	0.9991	1.1884	1.2492	1.3429	1.3006	1.4148	1.5962	1.3616	1.5088	1.7527
GLSAD (normal *h*)	1.0935	1.0821	1.0788	1.2024	1.1642	1.1863	1.2595	1.2364	1.2403	1.2906	1.2616	1.3289
GLSAD (*h*=0.5)	1.0614	1.0267	1.0033	1.0633	1.0722	1.0763	1.0917	1.1110	1.1356	1.1168	1.1319	1.1744
GLSAD (*h*=1.0)	1.0079	1.0125	0.9991	1.1168	1.1382	1.1840	1.1868	1.2291	1.3288	1.2255	1.2827	1.4209
GLSAD (*h*=1.5)	1.0065	1.0083	0.9989	1.1498	1.1864	1.2529	1.2391	1.3102	1.4454	1.2879	1.3821	1.5658
$\sigma_\mu^2 = 6$												
OLS	2.8671	4.2903	4.9044	3.3441	5.2463	6.2324	3.6114	5.8415	7.2000	3.7556	6.1800	7.7956
Within	1.0499	1.0130	1.0115	1.2280	1.2501	1.3581	1.3350	1.4114	1.6114	1.3932	1.5043	1.7675
GLSH_A	1.0043	1.0028	0.9995	1.1748	1.2426	1.3412	1.2774	1.4040	1.5917	1.3334	1.4968	1.7461
GLSH_WH	1.0056	1.0031	0.9994	1.1767	1.2433	1.3410	1.2797	1.4049	1.5915	1.3357	1.4978	1.7458
EGLS (normal *h*)	1.0068	1.0013	0.9998	1.1788	1.2443	1.3412	1.2824	1.4072	1.5916	1.3389	1.5008	1.7459
EGLS (*h*=0.5)	1.0028	1.0015	0.9999	1.1729	1.2419	1.3415	1.2756	1.4036	1.5919	1.3315	1.4966	1.7463
EGLS (*h*=1.0)	1.0038	1.0026	0.9996	1.1742	1.2426	1.3413	1.2768	1.4042	1.5917	1.3328	1.4970	1.7461
EGLS (*h*=1.5)	1.0042	1.0028	0.9995	1.1747	1.2427	1.3412	1.2774	1.4042	1.5917	1.3334	1.4971	1.7461
GLSAD (normal *h*)	1.4187	1.4078	1.5571	1.7160	2.1948	1.6732	1.9086	1.9800	1.9522	1.9286	2.0949	2.0591
GLSAD (*h*=0.5)	1.0739	1.0927	1.0191	1.1369	1.1464	1.1194	1.2021	1.2342	1.1862	1.2387	1.2995	1.2420
GLSAD (*h*=1.0)	1.0132	1.0236	1.0011	1.1188	1.1487	1.1871	1.1632	1.2374	1.3306	1.2187	1.2898	1.4216
GLSAD (*h*=1.5)	1.0079	1.0114	0.9994	1.1425	1.1880	1.2530	1.2249	1.3092	1.4436	1.2702	1.3796	1.5625

Table 6 - Percent of negative variances for EGLS and GLSAD estimators in case 2, *N*=50, 1000 replications

	λ											
	0			1			2			3		
	T=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10
$\sigma_{\mu}^{2} = 2$												
EGLS (normal *h*)	7.36	2.71	0.24	19.21	13.34	4.14	22.8	17.3	5.66	24.28	18.93	6.22
EGLS (*h*=0.5)	1.83	0.08	0	8.09	2.06	0	10.54	3.29	0.01	11.57	3.87	0.02
EGLS (*h*=1.0)	0.71	0	0	1.51	0.02	0	1.97	0.06	0	2.19	0.07	0
EGLS (*h*=1.5)	0.56	0	0	0.8	0	0	0.93	0	0	1.03	0	0
GLSAD (normal *h*)	0.01	0	0	0.22	0.09	0.05	0.51	0.28	0.27	0.7	0.49	0.49
GLSAD (*h*=0.5)	0	0	0	0	0	0	0.01	0	0	0.02	0	0
GLSAD (*h*=1.0)	0	0	0	0	0	0	0	0	0	0	0	0
GLSAD (*h*=1.5)	0	0	0	0	0	0	0	0	0	0	0	0
$\sigma_{\mu}^{2} = 4$												
EGLS (normal *h*)	0.2	0.12	0	1.14	0.57	0.07	1.69	0.98	0.12	2.05	1.19	0.16
EGLS (*h*=0.5)	0	0	0	0.05	0	0	0.1	0.01	0	0.13	0.01	0
EGLS (*h*=1.0)	0	0	0	0	0	0	0	0	0	0	0	0
EGLS (*h*=1.5)	0	0	0	0	0	0	0	0	0	0	0	0
GLSAD (normal *h*)	0.46	0.16	0.04	1.53	1.13	0.9	2.32	1.95	1.81	2.78	2.39	2.4
GLSAD (*h*=0.5)	0	0	0	0.12	0.03	0.01	0.26	0.13	0.04	0.35	0.21	0.08
GLSAD (*h*=1.0)	0	0	0	0	0	0	0	0	0	0	0	0
GLSAD (*h*=1.5)	0	0	0	0	0	0	0	0	0	0	0	0
$\sigma_{\mu}^{2} = 6$												
EGLS (normal *h*)	0	0	0	0.01	0	0	0.01	0	0	0.01	0	0
EGLS (*h*=0.5)	0	0	0	0	0	0	0	0	0	0	0	0
EGLS (*h*=1.0)	0	0	0	0	0	0	0	0	0	0	0	0
EGLS (*h*=1.5)	0	0	0	0	0	0	0	0	0	0	0	0
GLSAD (normal *h*)	6.39	4.02	1.66	8.64	6.68	4.84	9.70	8.23	6.34	10.21	9.06	7.03
GLSAD (*h*=0.5)	0.93	0.36	0. 04	2.21	1.52	0.74	2.83	2.11	1.27	3.15	2.45	1.57
GLSAD (*h*=1.0)	0	0	0	0.03	0.01	0	0.05	0.02	0	0.06	0.03	0
GLSAD (*h*=1.5)	0	0	0	0	0	0	0	0	0	0	0	0

Table 7 - 5% size performance of the *t*-ratio for β_1 in case 1, *N*=50, 1000 replications

	λ											
	0			1			2			3		
	T=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10
$\sigma_v^2 = 2$												
OLS	9.4	10.9	8.9	10.6	12.5	9.0	10.3	12.8	8.7	10.2	12.7	8.6
Within	4.8	4.7	5.3	4.8	4.7	5.3	4.8	4.7	5.3	4.8	4.7	5.3
GLSH_A	5.5	4.2	5.3	5.4	4.6	5.3	5.5	4.7	5.3	5.6	4.7	5.3
GLSH_WH	5.3	4.0	5.3	5.1	4.6	5.3	5.3	4.6	5.3	5.3	4.6	5.3
EGLS (normal *h*)	5.5	4.2	5.3	5.5	4.6	5.2	5.6	4.7	5.2	5.6	4.7	5.2
EGLS (*h*=0.5)	5.4	4.3	5.3	5.7	4.3	5.3	5.7	4.4	5.3	5.5	4.4	5.3
EGLS (*h*=1.0)	5.5	4.2	5.3	5.5	4.3	5.3	5.2	4.4	5.3	5.2	4.3	5.3
EGLS (*h*=1.5)	5.5	4.1	5.3	5.3	4.3	5.3	5.2	4.5	5.3	5.2	4.5	5.3
GLSAD (normal *h*)	11.5	12.3	14.8	11.7	15.0	17.2	13.5	17.5	17.9	13.9	17.6	17.1
GLSAD (*h*=0.5)	8.3	6.7	7.1	9.3	9.7	7.8	9.7	9.3	8.2	10.7	9.7	8.9
GLSAD (*h*=1.0)	6.0	4.7	5.3	7.3	5.6	5.1	7.5	7.1	5.1	7.9	7.3	5.2
GLSAD (*h*=1.5)	5.5	4.1	5.2	5.7	4.3	5.1	6.0	4.3	5.1	6.1	4.4	5.1
$\sigma_v^2 = 4$												
OLS	8.2	8.8	8.2	7.8	9.7	8.0	8.4	10.2	7.8	8.6	10.1	7.5
Within	4.8	4.7	5.3	4.8	4.7	5.3	4.8	4.7	5.3	4.8	4.7	5.3
GLSH_A	6.0	4.6	5.1	5.9	4.7	5.1	5.9	4.6	5.2	6.0	4.8	5.3
GLSH_WH	6.0	4.9	5.0	6.0	4.7	5.0	6.1	4.6	5.2	6.2	4.7	5.3
EGLS (normal *h*)	6.4	5.0	5.2	6.5	4.9	5.2	6.4	4.9	5.2	6.3	5.0	5.3
EGLS (*h*=0.5)	5.9	4.8	5.0	6.1	4.4	5.0	6.1	4.4	5.0	6.1	4.4	5.1
EGLS (*h*=1.0)	5.9	4.6	5.0	5.9	4.6	5.0	6.0	4.7	5.1	6.0	4.6	5.1
EGLS (*h*=1.5)	5.9	4.6	5.0	5.8	4.7	5.0	5.9	4.7	5.1	5.9	4.6	5.1
GLSAD (normal *h*)	9.0	7.3	7.4	9.0	8.2	7.9	9.6	8.6	8.2	9.8	8.6	8.2
GLSAD (*h*=0.5)	7.0	5.2	5.4	7.9	5.9	5.5	8.1	6.0	5.4	8.0	6.2	5.4
GLSAD (*h*=1.0)	6.0	4.8	5.1	6.4	4.8	5.2	6.6	5.0	5.4	6.6	5.0	5.2
GLSAD (*h*=1.5)	6.0	4.9	5.2	6.0	4.7	5.2	6.2	4.8	5.2	6.3	4.9	5.2
$\sigma_v^2 = 6$												
OLS	7.7	7.8	6.8	7.4	8.0	6.7	7.5	8.1	6.8	7.3	7.9	6.7
Within	4.8	4.7	5.3	4.8	4.7	5.3	4.8	4.7	5.3	4.8	4.7	5.3
GLSH_A	6.3	5.8	5.1	5.9	5.9	5.2	6.1	6.0	5.3	6.2	6.0	5.3
GLSH_WH	6.4	5.8	5.1	6.3	5.9	5.2	6.4	6.1	5.3	6.4	6.0	5.3
EGLS (normal *h*)	6.9	5.6	5.2	6.5	5.6	5.3	6.6	5.6	5.4	6.6	5.9	5.4
EGLS (*h*=0.5)	6.3	5.6	5.2	6.2	5.7	5.2	6.3	5.8	5.3	6.1	5.9	5.3
EGLS (*h*=1.0)	6.4	5.6	5.1	6.3	5.8	5.2	6.2	5.7	5.3	6.0	5.8	5.3
EGLS (*h*=1.5)	6.3	5.6	5.1	6.2	5.9	5.2	6.2	5.9	5.3	6.2	5.7	5.3
GLSAD (normal *h*)	8.4	6.2	5.7	8.4	6.7	5.7	8.7	7.0	5.7	8.8	7.1	5.6
GLSAD (*h*=0.5)	7.2	5.5	5.2	6.9	5.8	5.0	6.7	5.8	5.0	6.7	5.6	5.1
GLSAD (*h*=1.0)	6.4	5.7	5.0	6.5	5.9	5.1	6.6	6.0	5.0	6.6	5.9	5.0
GLSAD (*h*=1.5)	6.3	5.8	5.0	6.4	5.9	5.2	6.5	6.0	5.2	6.5	6.0	5.2

Table 8 - 5% size performance of the *t*-ratio for β_1 in case 2, *N*=50, 1000 replications

	λ											
	0			1			2			3		
	T=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10	*T*=3	*T*=5	*T*=10
$\sigma_\mu^2 = 2$												
OLS	7.7	7.8	6.8	7.9	9.6	7.1	8.3	10.5	7.3	8.6	10.7	7.7
Within	4.8	4.7	5.3	4.8	5.8	5.6	5.4	6.6	5.9	5.4	7.2	6.2
GLSH_A	6.3	5.8	5.1	7.2	7.6	5.8	7.5	7.9	6.7	7.8	8.3	6.7
GLSH_WH	6.4	5.8	5.1	7.1	7.8	5.8	7.4	7.9	6.6	7.7	8.3	6.8
EGLS (normal *h*)	6.9	5.6	5.2	7.2	7.0	6.6	7.6	8.0	7.1	7.5	8.1	7.4
EGLS (*h*=0.5)	6.3	5.6	5.2	6.8	7.5	5.8	7.5	8.1	6.5	7.8	8.5	6.6
EGLS (*h*=1.0)	6.4	5.6	5.1	7.2	7.7	5.8	7.4	8.1	6.6	7.9	8.4	6.8
EGLS (*h*=1.5)	6.3	5.6	5.1	7.2	7.7	5.8	7.3	7.9	6.6	7.6	8.5	6.8
GLSAD (normal *h*)	8.4	6.2	5.7	7.6	6.3	6.3	7.5	6.0	5.6	7.7	5.8	5.6
GLSAD (*h*=0.5)	7.2	5.5	5.2	6.8	5.6	4.8	6.1	5.6	4.5	6.0	5.4	4.4
GLSAD (*h*=1.0)	6.4	5.7	5.0	6.7	5.9	4.8	6.6	6.2	4.7	6.7	6.2	4.9
GLSAD (*h*=1.5)	6.3	5.8	5.0	6.5	7.1	5.2	7.0	6.9	5.1	7.2	6.9	5.2
$\sigma_\mu^2 = 4$												
OLS	8.2	8.8	8.2	8.9	10.0	7.9	9.2	10.3	8.6	9.4	10.6	9.0
Within	4.8	4.7	5.3	4.8	5.8	5.6	5.4	6.6	5.9	5.4	7.2	6.2
GLSH_A	6.0	4.6	5.1	6.7	7.1	6.1	6.9	7.6	6.8	6.7	7.6	7.0
GLSH_WH	6.0	4.9	5.0	6.9	6.9	6.1	6.8	7.5	6.7	6.7	7.6	7.0
EGLS (normal *h*)	6.4	5.0	5.2	6.7	6.5	6.0	7.2	7.6	6.5	7.1	7.5	6.8
EGLS (*h*=0.5)	5.9	4.8	5.0	6.5	7.0	6.1	6.9	7.5	6.6	6.7	7.6	6.8
EGLS (*h*=1.0)	5.9	4.6	5.0	6.6	6.8	6.0	6.5	7.5	6.6	6.5	7.5	6.9
EGLS (*h*=1.5)	5.9	4.6	5.0	6.6	6.8	5.9	6.5	7.4	6.6	6.5	7.5	6.9
GLSAD (normal *h*)	9.0	7.3	7.4	8.2	7.6	6.4	8.7	6.9	8.0	8.7	7.3	6.7
GLSAD (*h*=0.5)	7.0	5.2	5.4	7.1	6.3	5.4	6.5	5.6	5.2	6.3	5.4	4.5
GLSAD (*h*=1.0)	6.0	4.8	5.1	6.4	6.0	4.6	6.2	5.8	4.6	6.3	5.8	4.6
GLSAD (*h*=1.5)	6.0	4.9	5.2	6.3	6.5	5.3	6.4	6.7	5.2	6.2	6.9	5.4
$\sigma_\mu^2 = 6$												
OLS	9.4	10.9	8.9	10.2	11.3	9.4	10.3	11.1	9.6	10.7	11.3	9.4
Within	4.8	4.7	5.3	4.8	5.8	5.6	5.4	6.6	5.9	5.4	7.2	6.2
GLSH_A	5.5	4.2	5.3	5.8	6.6	6.0	6.2	7.4	6.4	6.3	7.6	6.9
GLSH_WH	5.3	4.0	5.3	5.7	6.5	6.0	6.0	7.0	6.4	6.3	7.4	6.7
EGLS (normal *h*)	5.5	4.2	5.3	5.9	6.7	6.2	6.3	7.1	6.6	6.3	7.6	6.8
EGLS (*h*=0.5)	5.4	4.3	5.3	5.7	6.5	6.1	6.2	7.1	6.4	6.3	7.5	6.8
EGLS (*h*=1.0)	5.5	4.2	5.3	5.6	6.6	6.0	6.1	7.1	6.3	6.3	7.5	6.7
EGLS (*h*=1.5)	5.5	4.1	5.3	5.6	6.6	6.0	6.1	7.1	6.3	6.3	7.5	6.7
GLSAD (normal *h*)	11.5	12.3	14.8	11.4	13.3	13.5	10.8	12.7	10.9	10.9	12.5	11.3
GLSAD (*h*=0.5)	8.3	6.7	7.1	7.7	6.5	6.9	7.6	6.5	6.3	7.2	6.4	6.6
GLSAD (*h*=1.0)	6.0	4.7	5.3	6.0	4.9	4.7	5.5	4.7	4.7	5.6	5.0	4.5
GLSAD (*h*=1.5)	5.5	4.1	5.2	5.5	5.3	5.3	5.8	5.9	4.9	5.9	5.8	5.3